Math 118 – General Education Mathematics
Formula Sheet

Logic

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Table 1: Truth Tables for conjunction, disjunction, conditional, and biconditional statements.

DeMorgan's Laws: $\sim(p \wedge q) \equiv \sim p \vee \sim q$
$\sim(p \vee q) \equiv \sim p \wedge \sim q$

Conditional as a disjunction: $p \rightarrow q \equiv \sim p \vee q$

Negation of Conditional: $\sim(p \rightarrow q) \equiv p \wedge \sim q$

Converse of $p \rightarrow q$: $q \rightarrow p$

Inverse of $p \rightarrow q$: $\sim p \rightarrow \sim q$

Contrapositive of $p \rightarrow q$: $\sim q \rightarrow \sim p$

Common Translations of $p \rightarrow q$:

If p, then q.	p is sufficient for q.
If p, q.	q is necessary for p.
p implies q.	All p are q.
p only if q.	q if p.

Valid argument forms:

Argument Form	Name	Argument Form	Name
$p \rightarrow q$ p $\therefore q$	Modus Ponens	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	Modus Tollens
$p \vee q$ $\sim p$ $\therefore q$	Disjunctive Syllogism	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical Syllogism

Invalid argument forms:

Argument Form	Name	Argument Form	Name
$p \rightarrow q$ q $\therefore p$	Fallacy of the Converse	$p \rightarrow q$ $\sim p$ $\therefore \sim q$	Fallacy of the Inverse

Counting

Number of Distinguishable Arrangements: $\frac{n!}{n_1!n_2!\cdots n_k!}$, where n is the total number of objects, and $n_1, n_2, \ldots, n_k$ are the number of objects of type $1, 2, \ldots, k$ with $n_1 + n_2 + \cdots + n_k = n$.

Permutation of n objects taken r at a time: ${}_nP_r = \frac{n!}{(n-r)!}$

Combination of n objects taken r at a time: ${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

Permutation/Combination Relation: ${}_nP_r = r! \cdot {}_nC_r$

Deck of Cards Breakdown: 52 cards total, with four (13 card) suits (red: ♡, ♢; black: ♣, ♠). Cards per suit: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King

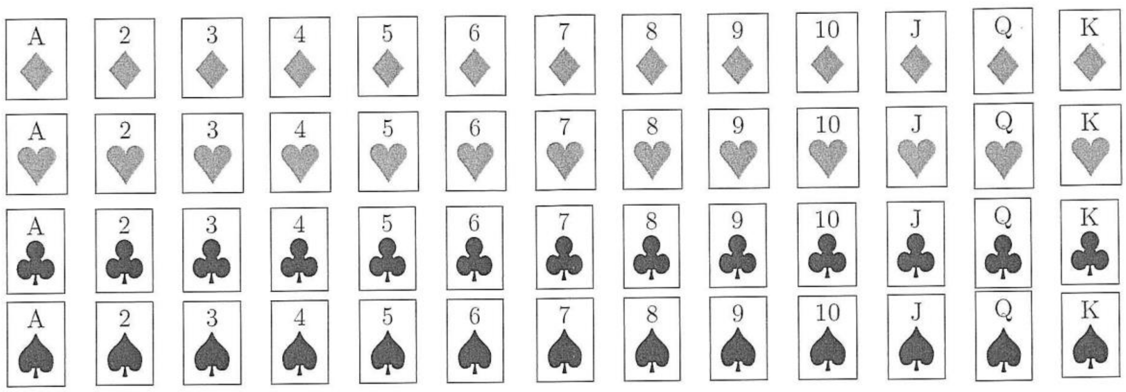

Table 2: The layout of a standard deck of cards.

Complement property: $n(A') = n(\mathcal{U}) - n(A)$, where $\mathcal{U}$ is the universal set and $A \subseteq \mathcal{U}$.

Addition Property: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Pascal's Triangle:

Row Number		Row Sum
0	1	$2^0 = 1$
1	1 1	$2^1 = 2$
2	1 2 1	$2^2 = 4$
3	1 3 3 1	$2^3 = 8$
4	1 4 6 4 1	$2^4 = 16$
5	1 5 10 10 5 1	$2^5 = 32$
6	1 6 15 20 15 6 1	$2^6 = 64$
7	1 7 21 35 35 21 7 1	$2^7 = 128$
8	1 8 28 56 70 56 28 8 1	$2^8 = 256$
9	1 9 36 84 126 126 84 36 9 1	$2^9 = 512$
10	1 10 45 120 210 252 210 120 45 10 1	$2^{10} = 1024$

Figure 1: Pascal's Triangle

Probability

Theoretical probability: $P(E) = \dfrac{\text{number of times event } E \text{ occurs}}{\text{total number of outcomes}} = \dfrac{n(E)}{n(S)}$, where S is the sample space, and $E \subseteq S$ is an event in the sample space.

Empirical Probability: $P(E) = \dfrac{\text{number of times event } E \text{ occurs}}{\text{total number of observations/trials}}$

General Properties: $0 \leq P(E) \leq 1$; $P(E) = 0$ is an impossible event (i.e. $E = \emptyset$); $P(E) = 1$ is a certain event (i.e. $E = S$).

Complement property: $P(A') = 1 - P(A)$

Addition Property: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; alternatively, $P(A \cup B) = P(A) + P(B)$ if and only if A and B are **mutually exclusive** (i.e. $A \cap B = \emptyset$).

Conditional Probability: $P(A \mid B) = \dfrac{n(A \cap B)}{n(B)} = \dfrac{P(A \cap B)}{P(B)}$ and $P(B \mid A) = \dfrac{n(A \cap B)}{n(A)} = \dfrac{P(A \cap B)}{P(A)}$.

Multiplication Rule: $P(A \cap B) = P(A \mid B)P(B) = P(B \mid A)P(A)$. Events A and B are said to be **independent** if $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$. Therefore, $P(A \cap B) = P(A) \cdot P(B)$ when A and B are independent.

Probability Distribution Properties: (i) $0 \leq P(x) \leq 1$ for each x value; (ii) $\sum P(x) = 1$ (the sum of all the probabilities must be equal to 1).

Binomial Distribution Function: $f(x) = {}_nC_x \cdot p^x \cdot (1-p)^{n-x}$, where n is the number of trials, p is the probability of a success, and x is the number of successes. Recall that ${}_nC_x = \dfrac{n!}{x!(n-x)!}$.

Expected Value: $E(x) = \sum x \cdot P(x) = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_n P(x_n)$.

Expected Value for Binomial Distribution: If x is a binomial random variable, n is the number of trials of the binomial experiment, and p is the probability of success, then $E(x) = np$.

Statistics

Guidelines for Constructing Grouped Frequency Distributions:

1. Make sure each data item will fit into one, and only one, class.
2. Try to make all the classes the same width.
3. **Make sure the classes do not overlap.**
4. Use from 5 to 12 classes.

Mean: $\overline{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n} = \dfrac{\sum x}{n}$

Weighted Mean: $\overline{w} = \dfrac{w_1 \cdot x_1 + w_2 \cdot x_2 + \cdots + w_n \cdot x_n}{w_1 + w_2 + \cdots + w_n} = \dfrac{\sum w \cdot x}{\sum w}$

Position of the Median of Data Set of Size n: $\dfrac{n+1}{2}$

Range of Data Set: maximum value − minimum value.

Standard Deviation: $s = \sqrt{\dfrac{\sum(x-\overline{x})^2}{n-1}}$ or $s = \sqrt{\dfrac{n\sum x^2 - (\sum x)^2}{n(n-1)}}$.

Chebyshev's Theorem: At least $1 - \dfrac{1}{k^2}$ of the data lies within k S.D. of the mean, where $k > 1$

Coefficient of Variation: $V = \dfrac{s}{\overline{x}} \times 100$ (for samples); $V = \dfrac{\sigma}{\mu} \times 100$ (for populations).

Quartiles: Q_1 is the median of the first half of the data set, Q_2 is the median of the data set, Q_3 is the median of the second half of the data set.

Five Number Summary: The five values used to construct box plots are: Min, Q_1, Q_2, Q_3, Max.

Empirical Rule: For normally distributed data, 68% of the data lies within one S.D. of the mean, 95% of the data lies within 2 S.D. of the mean, and 99.7% of the data lies within 3 S.D. of the mean.

z-score: $z = \dfrac{x - \overline{x}}{s}$ or $z = \dfrac{x - \mu}{\sigma}$

Linear Regression: Given data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, the equation of the regression line is $y' = ax + b$ where

$$a = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2} \quad \text{and} \quad b = \frac{\sum y - a\left(\sum x\right)}{n}$$

Correlation Coefficient: $r = \dfrac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{\left[n\left(\sum x^2\right) - \left(\sum x\right)^2\right]\left[n\left(\sum y^2\right) - \left(\sum y\right)^2\right]}}$

Interpreting r: If $r = \pm 1$, y' is a perfect fit. If r is close to 1 or -1, the line y' is a strong fit. If r is not close to 0 and not close to -1 or 1, y' is a moderate fit. If $r = 0$ or close to 0, it is a weak fit.