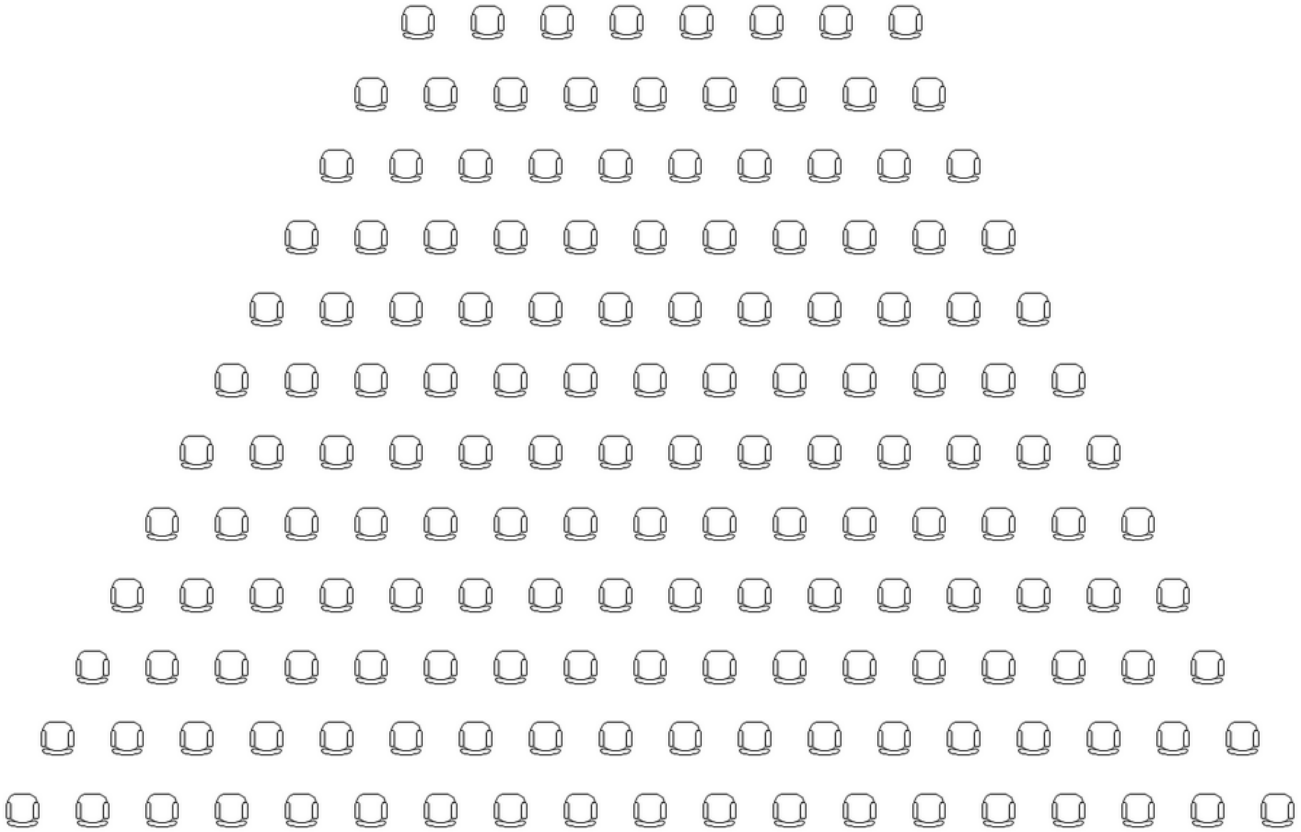
STAGE



THEATER Seats

There are 12 rows of seats in a small theater with 8 seats in the first row. Each higher row has one more seat than the row in front of it. In other words the second row has 9 seats, ..., and the last row has 19 seats.

(1) How many seats are there in the theater?

(2) What if you had something other than 8 seats in the first row, but you still had only 12 rows. Can you come up with a formula for the total number of seats if you start with "s" seats in the first row?

(3) What if you had something other than 12 rows. could you come up with a formula for "r" rows?

(4) What if you had "r" rows and "s" seats in the first row, now how many seats do you have in the theater.

Answers.

1 - You could count all of them and find that there are 162 seats. Or you could add up the number of seats in each of the 12 rows: 8+9+10+11+12+13+14+15+16+17+18+19 = 162.
If you find it hard to add up these twelve numbers, you might find some easier way to do it. For example, each row has 8 seats in it plus 0,1,2,3,4,5,6,7,8,9,10,11. you might find it easier to add these up, so the total is
0+1+2+3+4+5+6+7+8+9+10+11 = 66, plus 8 × 12= 96 which gives you the answer of 66 + 196 = 162
If you want another way to look at it, think of the same seats reversed with the first row having 19 seats and the last row having 8. Put these seats next to the original seats. Each row now has the same number of seats in it, since 8+19 = 9+18 = 10+17 = 27. So 12 rows with 27 seats equal 324 seats. but since is twice as many seats as we started with, the original count is 324 / 2 = 162. Or, put more succinctly. the total number of seats is equal to the number of seats in the first row plus the number of seats in the last row all times 12 divided by 2 giving you (8+19) * 12 / 2= 162

2 - Suppose you had 1 seat in the first row, then you would have 12 seats in row 12. Look at a few more examples, first row = 2 seats, last row will have 11 more seats, or 13 seats. In other words the last row will always have 11 more seats than the first row. So if the first row has s seats then the last row will have s+11 seats. Using the technique we found in problem one, the total number of seats = (s + (s+11)) * 12 / 2 Or if you had a friend who can do algebra, she will tell you that the answer is: (s + (s+11)) × 12 / 2 = 12 × s +66
You might want to go back and verify that this formula gives you the same answer we got for question (1) with s = 8. So 12 × s + 66 = 12 × 8 + 66 = 96 + 66 = 162

3 - If the first row has 8 seats, then last row has (8 plus r - 1) seats. Using the summation technique we get that the total number of seats is: (8 + (8 + r -1)) × r / 2 = (r^2 +15 r) / 2
Let's go back and verify that this formula also works for our original 12 rows: (r^2 +15 r) / 2 = (12*12 + 15*12) / 2= (144 + 180) / 2= (324) / 2= 162

4 - Combining the answers in questions (2) and (3) we get:
first row has s seats, last row has (s+r-1) seats and the total number of seats is
(s + (s+r-1)) * r / 2 = (r^2 +2rs -r) /2 seats.
Let's go check this formula: (12*12 + 2*12*8 - 12)/2 = (144 + 192 - 12)/2 = 324/2 = 162 seats.

And that is why algebra is useful.